Deterministically Preserve Seismic Amplitudes by Modeling Generalized Wave Field with Full Inhomogeneity and Full Anisotropy

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Full inhomogeneity: model with both vertical and lateral variation

Full anisotropy: model with 21 independent elastic moduli

Objective

Cook one more dish for

Amplitude Standard Project

Seismic Attributes

Waveform Modeling

Isotropic

Anisotropy

Decoupled: VTI

Coupled: ***orthorhombic & generic***

Goal of Modeling

Make a fancy dish: Cook all seismic waves together, and then extract the primaries deterministically to preserve seismic amplitudes.

Examples of seismic waves to be cooked:

- Ground rolls (surface waves)
- Guided waves
- Multiples
- Primaries
- Converted waves
- Decoupled seismic waves (in case of VTI)
- Coupled seismic waves (greater than 5 independent moduli).

Can we make it?

Yes. By using Generalized Matrix Element.

References:

J. H. Woodhouse, The Joint Inversion of Seismic Wave Forms for Lateral Variations in Earth Structure and Earthquake Source Parameters, Tipografia Compositori Bologna, 1983

Please bear me for a quick review of the methodology of

Generalized Matrix Element

for seismic wave modeling.

All seismic waves are imbedded in the following two equations:

•Generalized Hooke's Law (constitutive law or rheological law)

$$\begin{bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{33} \\ \tau_{12} \\ \tau_{13} \\ \tau_{23} \end{bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1112} & C_{1113} & C_{1123} \\ C_{1122} & C_{2222} & C_{2233} & C_{2212} & C_{2213} & C_{2223} \\ C_{1133} & C_{2233} & C_{3333} & C_{3312} & C_{3313} & C_{3323} \\ C_{1112} & C_{2212} & C_{3312} & C_{1212} & C_{1213} & C_{1223} \\ C_{1113} & C_{2213} & C_{3313} & C_{1213} & C_{1313} & C_{1323} \\ C_{1123} & C_{2223} & C_{3323} & C_{1223} & C_{1323} & C_{2323} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$

$$\tau_{ij} = C_{ijkl}\varepsilon_{kl} \quad \text{(Einstein Summation)}$$

•Conservation of Momentum (Equation of Motion)

$$\frac{\partial}{\partial t}\iiint_V \rho \frac{\partial \vec{u}}{\partial t} dV = \oiint_s \vec{T} ds + \iiint_V \vec{f} dV$$

$$\rho \ddot{u}_i = \tau_{ij,j} + f_i \quad \text{(Einstein Summation)}$$

Sub Hooke's law into Equation of Motion:

$$\rho \ddot{u}_i - C_{ijkl} \frac{\partial^2}{\partial x_j \partial x_l} u_k = f_i$$

Matrix Form:

$$\begin{bmatrix} \rho \frac{\partial^2}{\partial t^2} & 0 & 0 \\ 0 & \rho \frac{\partial^2}{\partial t^2} & 0 \\ 0 & 0 & \rho \frac{\partial^2}{\partial t^2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} - \begin{bmatrix} C_{1j1l} \frac{\partial^2}{x_j x_l}, & C_{1j2l} \frac{\partial^2}{x_j x_l}, & C_{1j3l} \frac{\partial^2}{x_j x_l} \\ C_{2j1l} \frac{\partial^2}{x_j x_l}, & C_{2j2l} \frac{\partial^2}{x_j x_l}, & C_{2j3l} \frac{\partial^2}{x_j x_l} \\ C_{3j1l} \frac{\partial^2}{x_j x_l}, & C_{3j2l} \frac{\partial^2}{x_j x_l}, & C_{3j3l} \frac{\partial^2}{x_j x_l} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

Define operators of Generalized Matrix Element:

$$
\begin{cases}
P \equiv \begin{bmatrix} \rho & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \rho \end{bmatrix} \\
G_{ik} \equiv C_{ijkl} \dfrac{\partial^2}{\partial x_j \partial x_l}
\end{cases}
$$

Equation of Motion becomes:

$$
P \frac{\partial^2}{\partial t^2} \vec{u} - G\vec{u} = \vec{f}
$$

De-power the differential equation in time:

$$\frac{\partial^2}{\partial t^2}\vec{u} = P^{-1}\vec{f} + P^{-1}G\vec{u}$$

$$\frac{\partial \vec{u}}{\partial t} = \frac{\partial \vec{u}}{\partial t}$$

Add-up equation

$$\frac{\partial}{\partial t}\begin{bmatrix} \vec{u} \\ \frac{\partial \vec{u}}{\partial t} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ P^{-1}G & 0 \end{bmatrix}\begin{bmatrix} \vec{u} \\ \frac{\partial \vec{u}}{\partial t} \end{bmatrix} + \begin{bmatrix} 0 \\ P^{-1}\vec{f} \end{bmatrix}$$

$$\frac{\partial}{\partial t}\vec{b} = A\vec{b} + \vec{g}$$

$$\vec{b} \equiv \begin{bmatrix} \vec{u} \\ \frac{\partial \vec{u}}{\partial t} \end{bmatrix} \qquad A \equiv \begin{bmatrix} 0 & 1 \\ P^{-1}G & 0 \end{bmatrix} \qquad \vec{g} \equiv \begin{bmatrix} 0 \\ P^{-1}\vec{f} \end{bmatrix}$$

Solution for vector ***b*** is handy

$$\vec{b}(t) = \int_0^t e^{A(t-t')}\vec{g}\,dt'$$

Using Propagator Matrix idea by Taylor Expansion:

$$e^{A(t-t')} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -x^2 & 0 \end{bmatrix}(t-t') + \frac{(t-t')^2}{2!}\begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix} + \begin{bmatrix} 0 & -x^2 \\ x^4 & 0 \end{bmatrix}\frac{(t-t')^3}{3!} + \begin{bmatrix} x^4 & 0 \\ 0 & x^4 \end{bmatrix}\frac{(t-t')^4}{4!} + \cdots\cdots$$

$$= \begin{bmatrix} \cos[(t-t')x] & x^{-1}\sin[(t-t')x] \\ -x\sin[(t-t')x] & \cos[(t-t')x] \end{bmatrix}$$

where

$$x^2 \equiv -p^{-1}G$$

Finally we get full seismic wave field:

$$\vec{u} = \int_0^t x^{-1}\left[\sin\left(\left(t - t'\right)x\right)\right]P^{-1}\vec{f}dt'$$

$$x^2 = -P^{-1}G$$

For exploration seismology, we need to do local representation for the wave field as

$$u_k = \sum_{m=1}^{M} V_{km}(t)\phi_m(\vec{x})$$

where

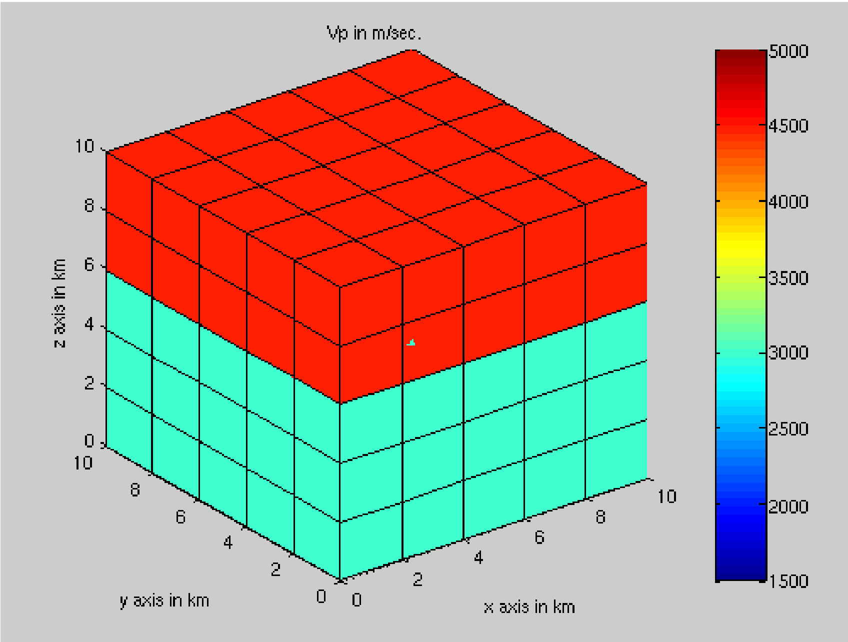
$$\phi_m(\vec{x}) = \phi_m(x_1, x_2, x_3)$$

is the local basis function.

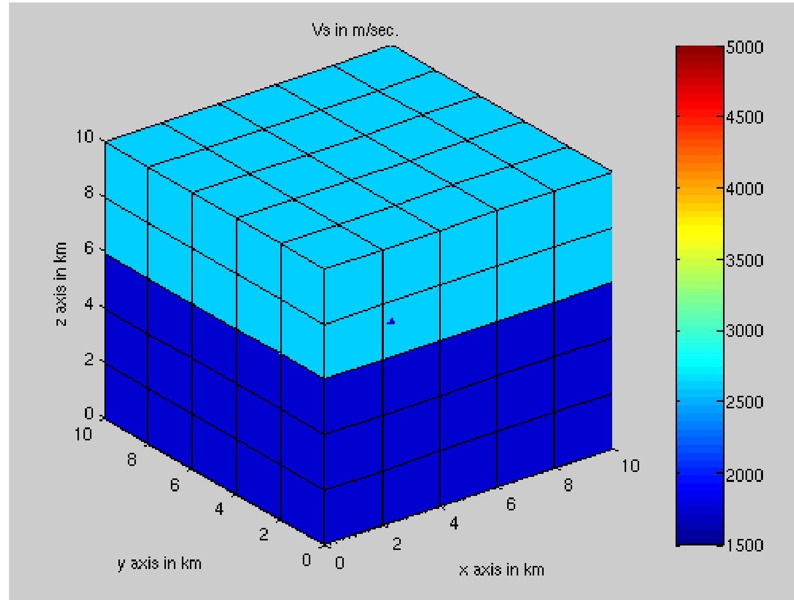
So much for the math!!! Let's see examples

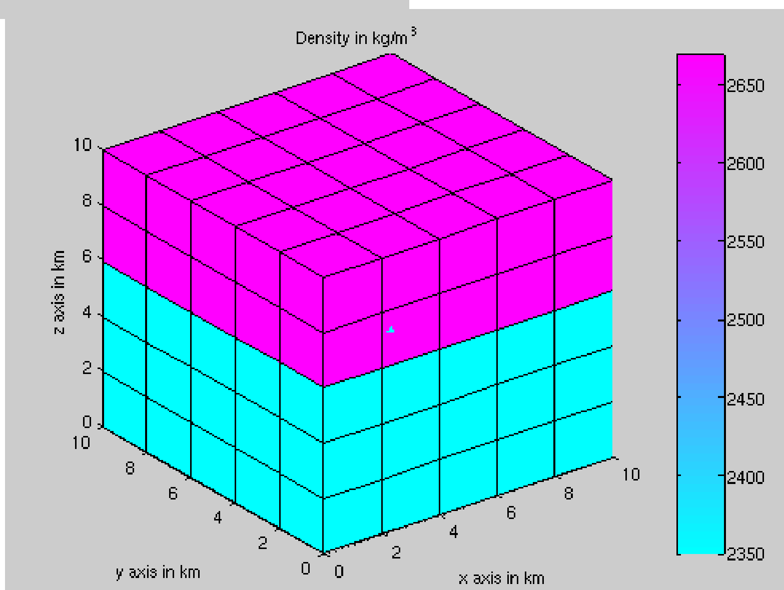
Isotropic Model No. 1 (w1_max=0.51 Hz, w2_max=0.51 Hz, w3_max=0.51 Hz)

P velocity



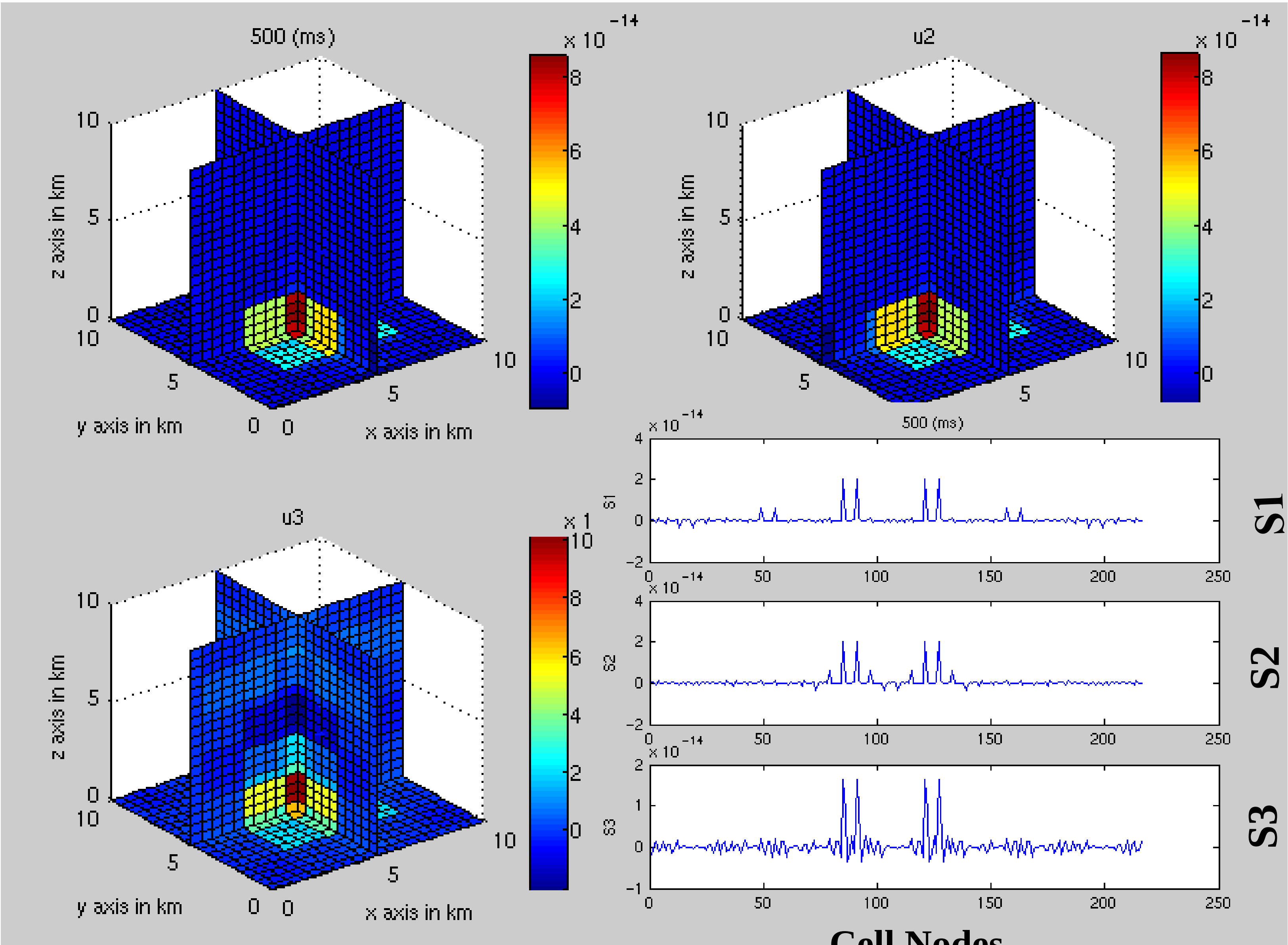
S velocity



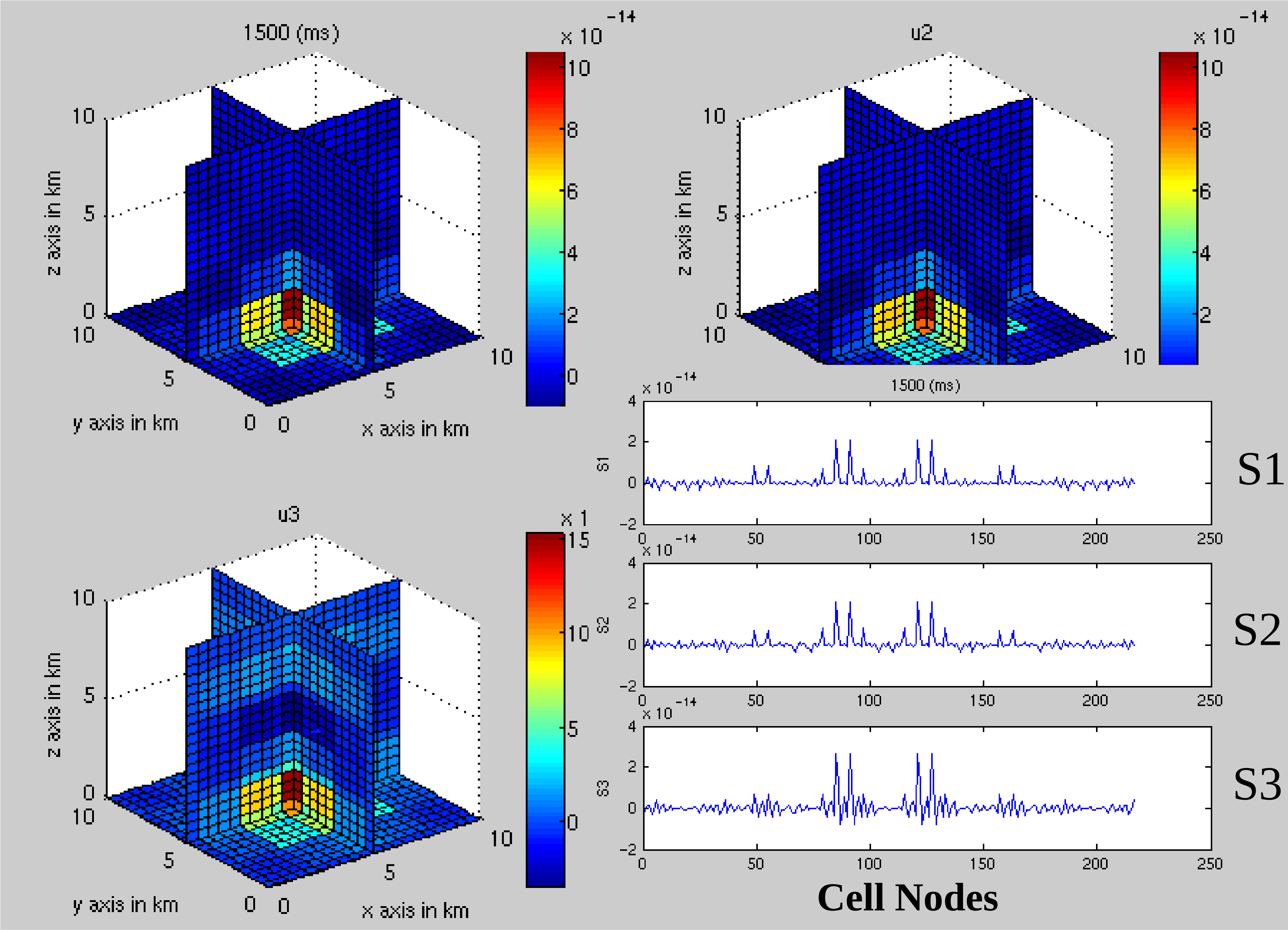


Density

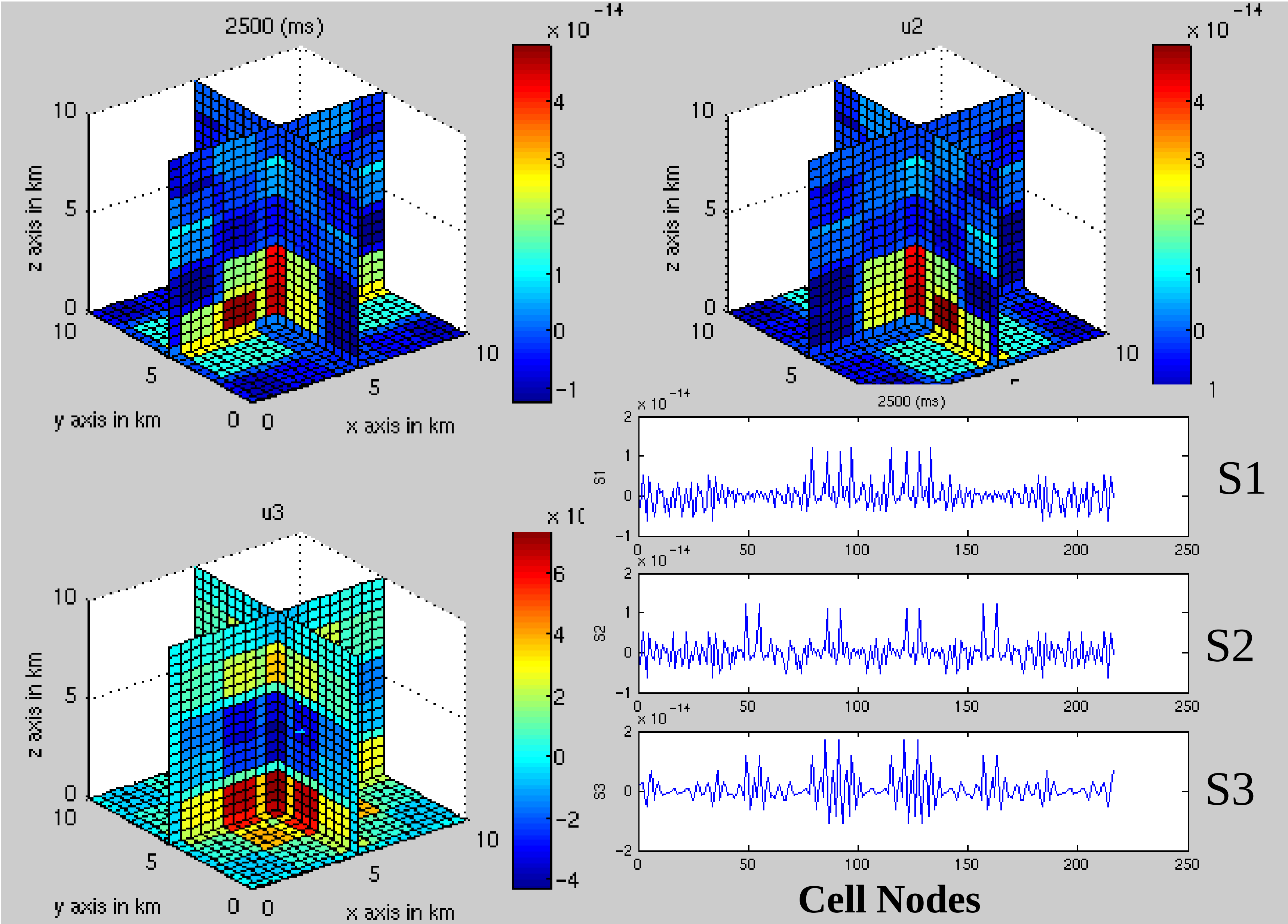
Displacements and Eigen Functions at 500 ms



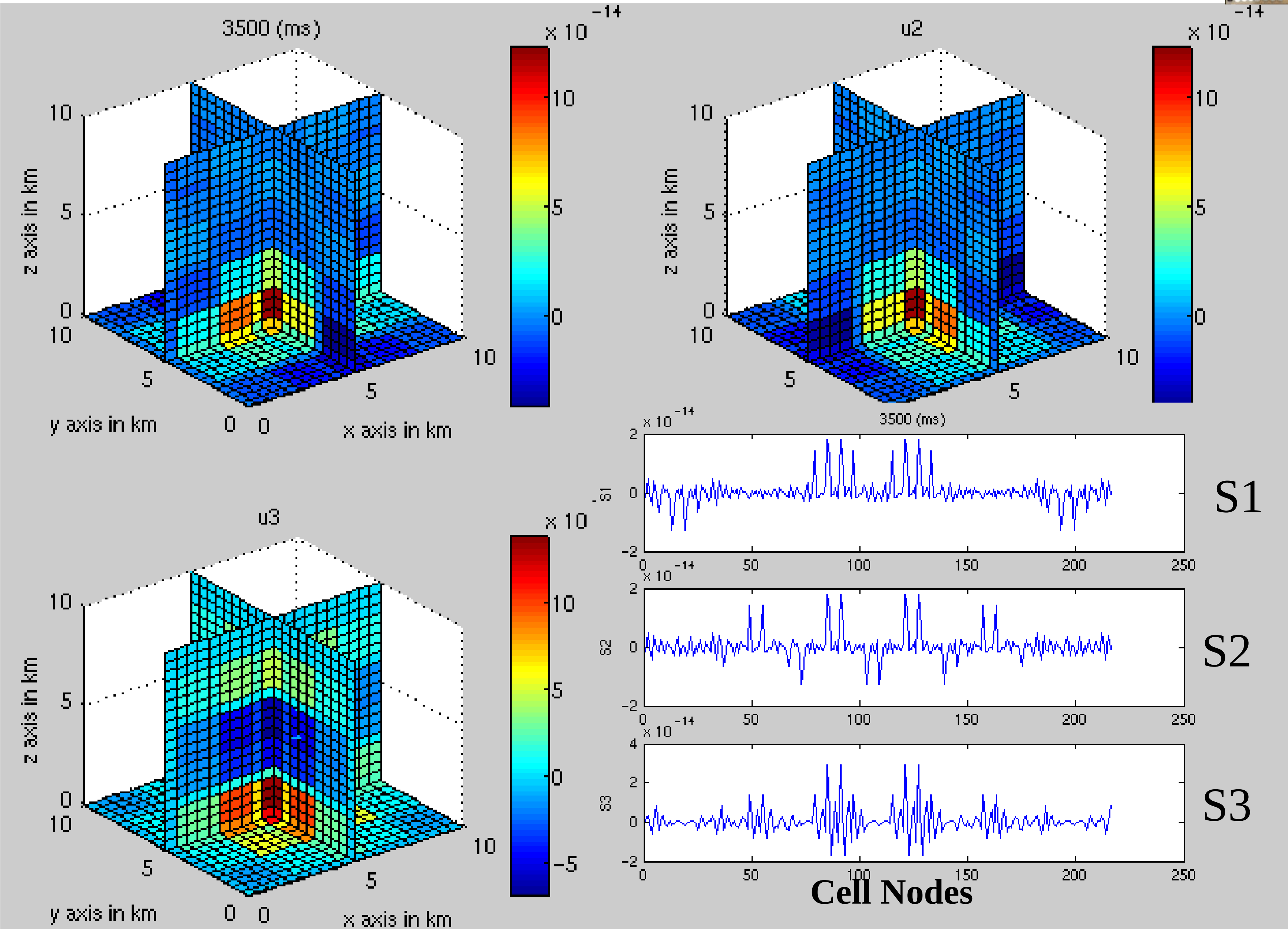
Displacements and Eigen Functions at 1500 ms



Displacements and Eigen Functions at 2500 ms

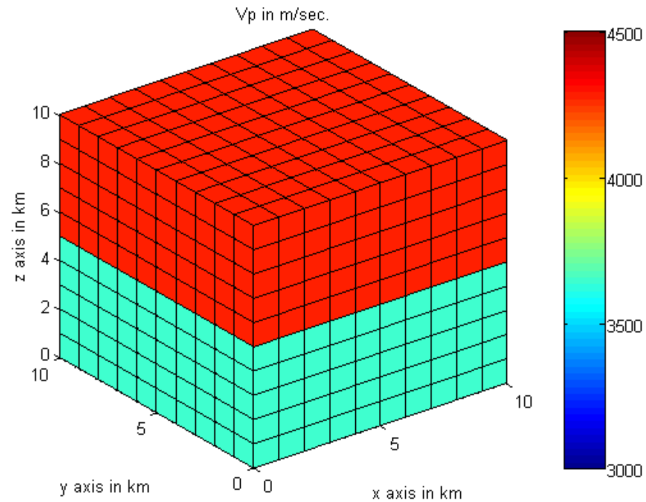


Displacements and Eigen Functions at 3500 ms

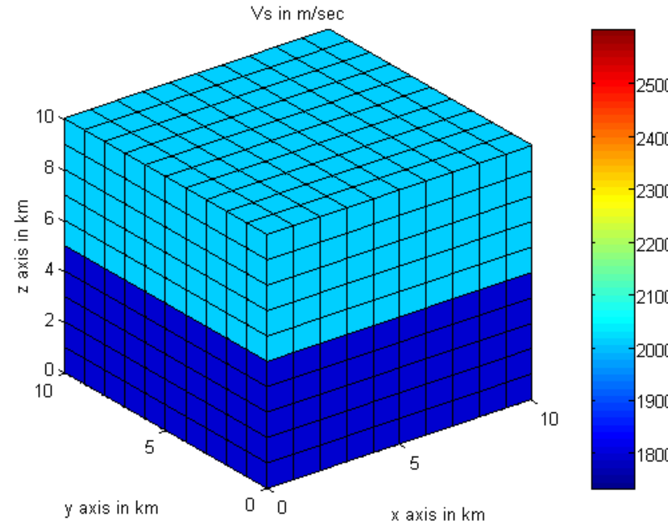


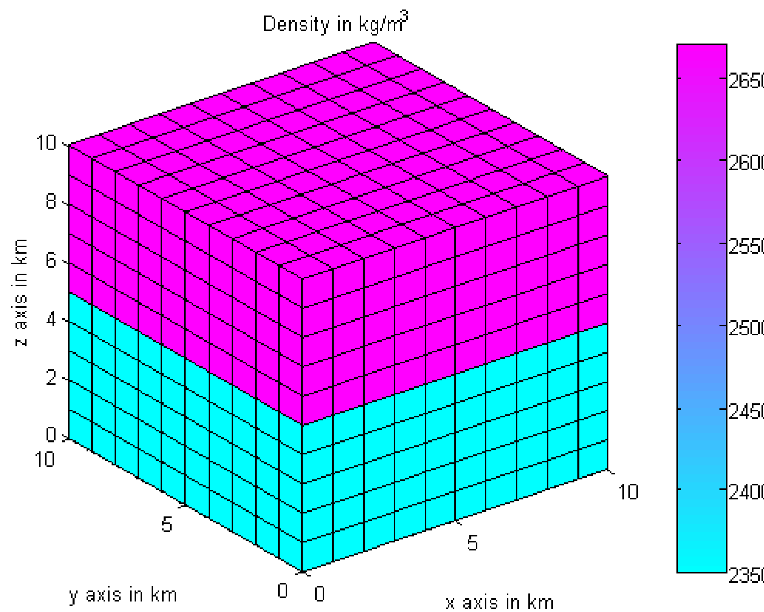
Isotropic Model 2 with a lot more cells

P velocity

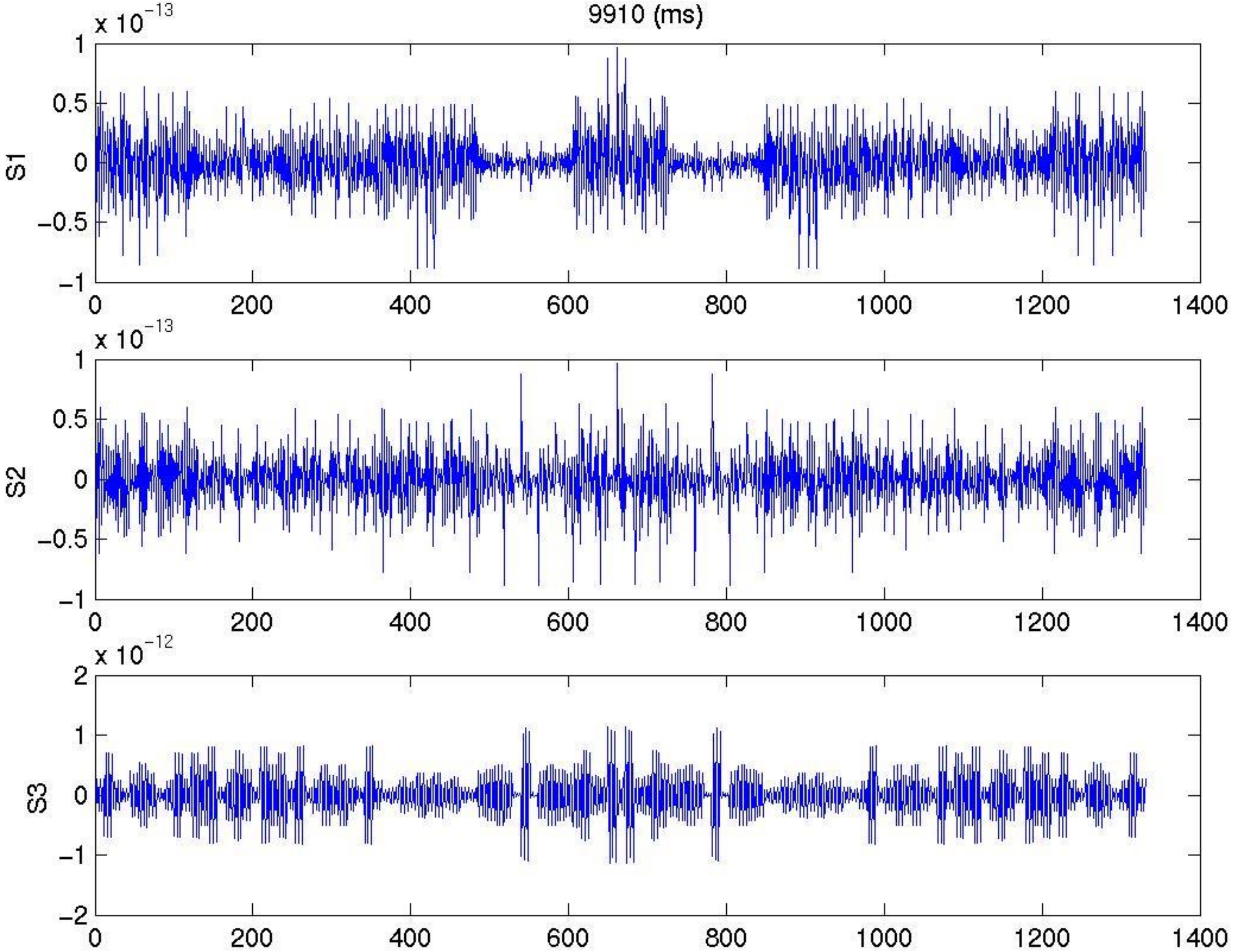


S velocity



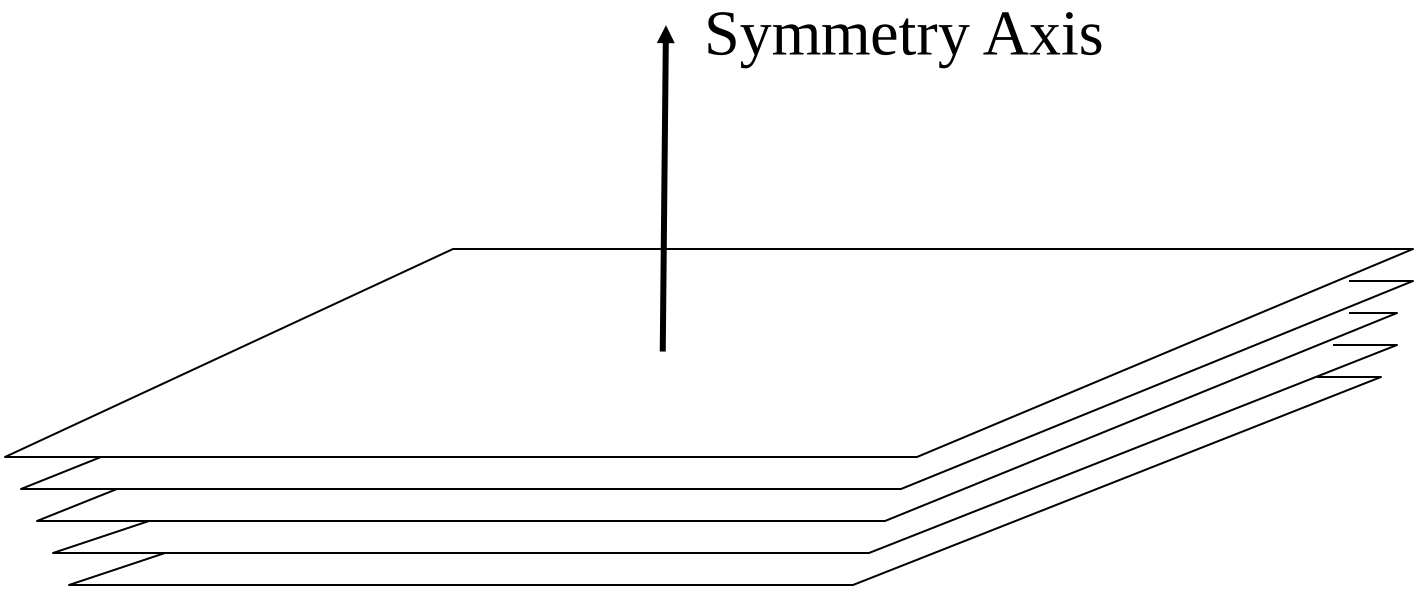


Density



3D cell nodes

Vertical Transverse Isotropy (five independent moduli) Corresponds to flat sediments with no deformation or complicated sedimentary patterns



Vertical Transverse Isotropy (five independent moduli)

$$\begin{bmatrix}\tau_{11}\\ \tau_{22}\\ \tau_{33}\\ \tau_{12}\\ \tau_{13}\\ \tau_{23}\end{bmatrix}=\begin{bmatrix}\lambda'+2\mu' & \lambda' & \lambda & 0 & 0 & 0\\ \lambda' & \lambda'+2\mu' & \lambda & 0 & 0 & 0\\ \lambda & \lambda & v & 0 & 0 & 0\\ 0 & 0 & 0 & \mu' & 0 & 0\\ 0 & 0 & 0 & 0 & \mu & 0\\ 0 & 0 & 0 & 0 & 0 & \mu\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{12}\\ 2\varepsilon_{13}\\ 2\varepsilon_{23}\end{bmatrix}=\begin{bmatrix}C_{1111} & C_{1122} & C_{1133} & 0 & 0 & 0\\ C_{2211} & C_{2222} & C_{2233} & 0 & 0 & 0\\ C_{3311} & C_{3322} & C_{3333} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{1212} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{1313} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{2323}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{12}\\ 2\varepsilon_{13}\\ 2\varepsilon_{23}\end{bmatrix}$$

Love's Notation

$$\rho V_{PH}^2 = C_{1111} = C_{2222} = \lambda' + 2\mu'$$

$$\rho V_{PV}^2 = C_{3333} = v$$

$$\rho V_{SH}^2 = C_{1212} = \mu'$$

$$\rho V_{SV}^2 = C_{1313} = C_{2323} = \mu$$

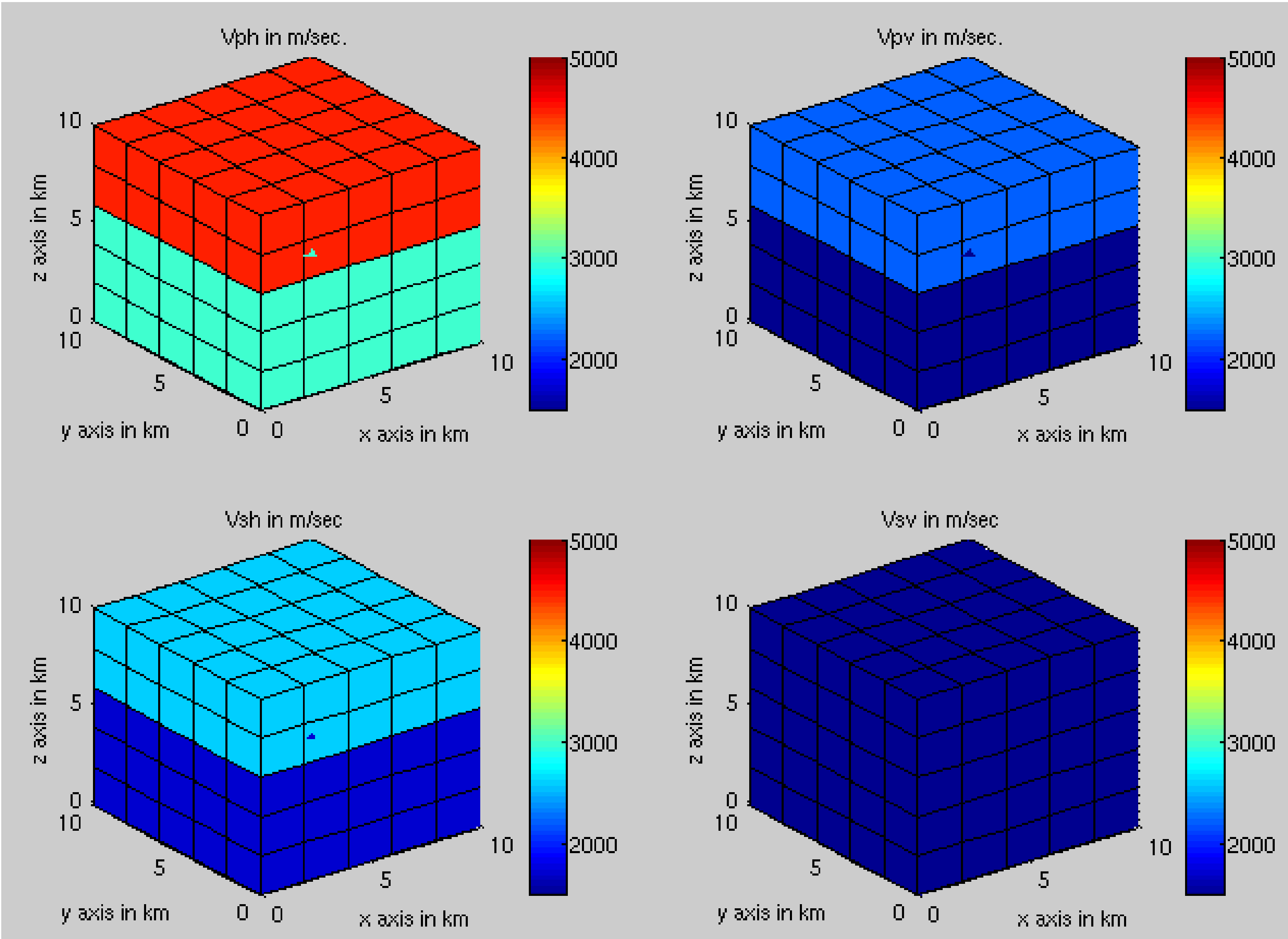
$$\eta\rho(V_{PH}^2 - 2V_{SV}^2) = C_{1133} = C_{3311} = C_{2233} = C_{3322} = \lambda$$

Assuming Poisson's medium:

$$\lambda' = \mu' = C_{1122} = C_{2211}$$

Velocity Field of Vertical Transverse Isotropy (Vsh > Vsv)

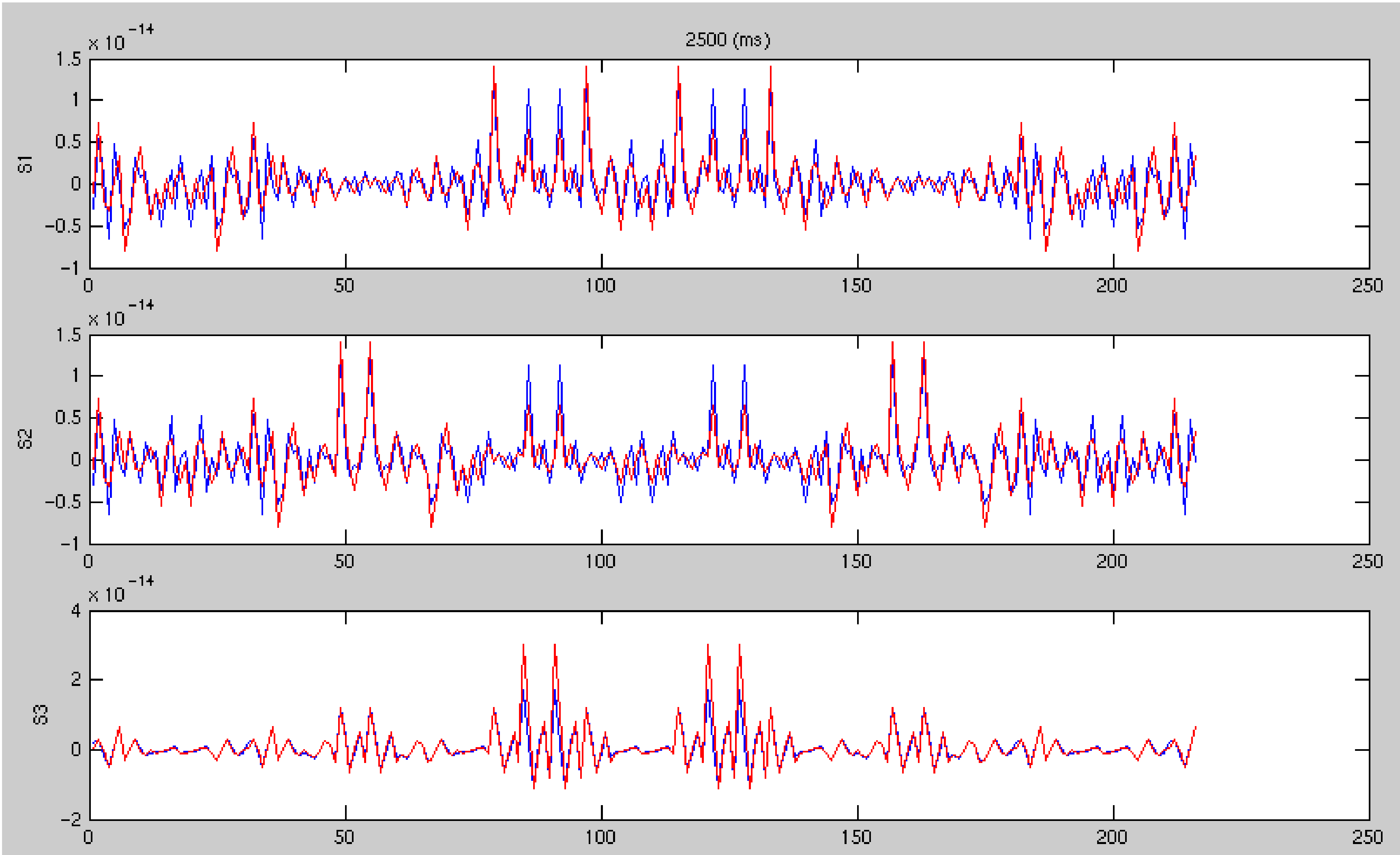
(w1_max=0.51 Hz, w2_max=0.51 Hz, w3_max=0.39 Hz, 25% Anisotropy)



Anisotropy(25% VTI) vs. Isotropy at 2500 ms

Eigenfunctions

S1
S2
S3



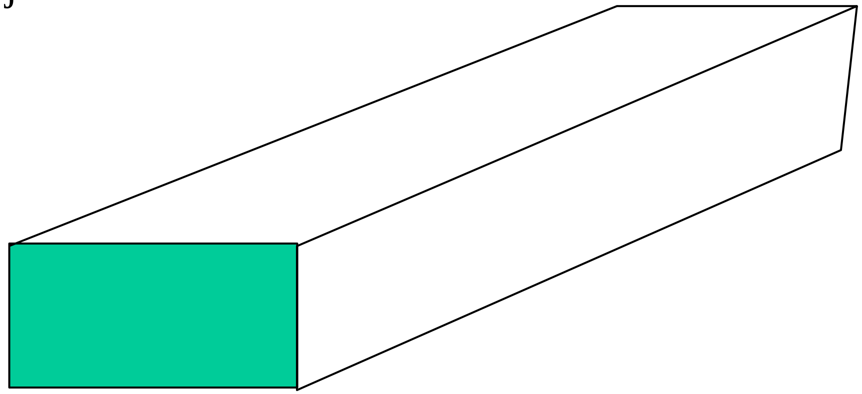
Blue=isotropy, Red=VTI

Cell nodes

Orthorhombic Anisotropy from phenolite
9 independent moduli
(simplest azimuth anisotropy)

$$
=\begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & 0 & 0 & 0 \\ C_{2211} & C_{2222} & C_{2233} & 0 & 0 & 0 \\ C_{3311} & C_{3322} & C_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{1212} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{2323} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix} = \begin{bmatrix} 20.80 & 11.47 & 7.26 & 0 & 0 & 0 \\ 11.47 & 17.46 & 7.87 & 0 & 0 & 0 \\ 7.26 & 7.87 & 10.06 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.59 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.59 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.04 \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}
$$

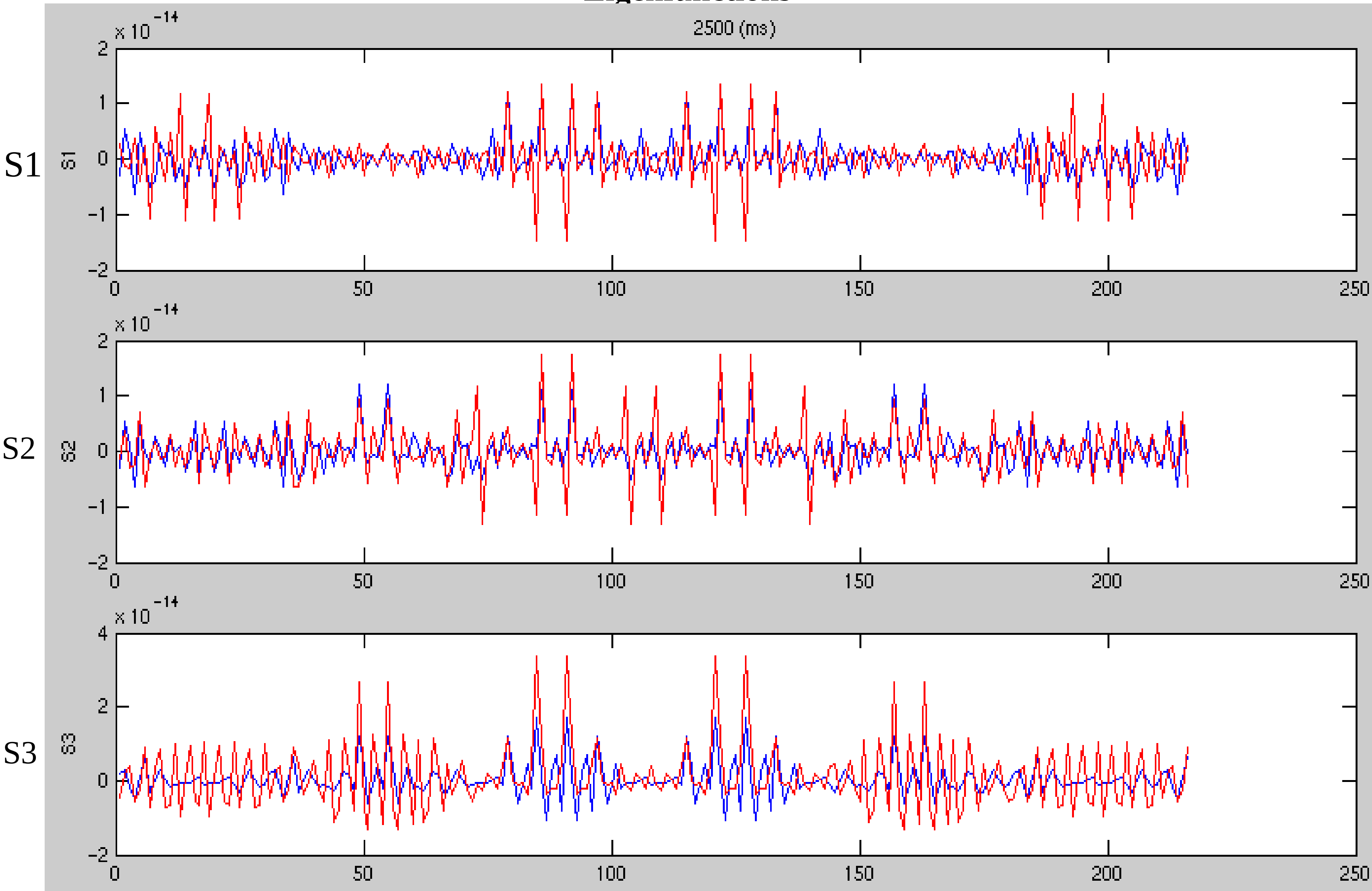
C_{ijkl} is in GPa



- Fracture cracks
- Sedimentary flows
- Stress induced anisotropy

Azimuth Anisotropy vs. Isotropy at 2500 ms

Eigenfunctions



Cell nodes

Blue=isotropy, **Red=azimuth**

Observed vs. Synthetic Seismograms